Plasma dispersion of multisubband electron systems over liquid helium

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Density-density response functions are evaluated for nondegenerate multisubband electron systems in the random-phase approximation for arbitrary wave number and subband index. We consider both quasi-two-dimensional and quasi-one-dimensional systems for electrons confined to the surface of liquid helium. The dispersion relations of longitudinal intrasubband and transverse intersubband modes are calculated at low temperatures and for long wavelengths. We discuss the effects of screening and two-subband occupancy on the plasmon spectrum. The characteristic absorption edge of the intersubband modes is shifted relatively to the single-particle intersubband separation and the depolarization shift correction can be significant at high electron densities.

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I. INTRODUCTION

Two-dimensional electron systems (2DES) over the liquid helium surface have been intensively studied for a long time. [1] More recently, it was possible to confine these surface electrons (SE) in reduced geometries creating also one-dimensional (1D) systems. [2]. Both

systems provide a nearly ideal laboratory for studying collective phenomena in the electron plasma in lower dimensions because the cleanness of the helium surface restricts SE scattering mechanisms to those with helium atoms in the vapor phase, which predominates at $T > 1$ K, and with surface oscillations (ripplons) at lower temperatures. Furthermore both scattering mechanisms become ineffective with lowering temperature and can be discarded at $T \lesssim 0.1$ K. In such a regime, collective effects in low-dimensional electron systems due to the Coulomb interaction can be investigated ignoring the interaction with scatterers. Another important feature of these systems is the accessible range of SE densities which is limited to $n_s \lesssim 10^9$ cm^{-2} (for bulk helium). As a consequence, the 2D Fermi energy $\varepsilon_F \lesssim 10^{-2}$ K and SE behave like nondegenerate low-dimensional systems differing in many aspects from its quantum counterpart realized in semiconductor structures. [3]

As it is well known, the collective excitation spectrum depends crucially on the way the particles are confined. For instance, for longitudinal plasma oscillations of the 2DES, the spectrum $\omega_{2D}(q) \sim q^{1/2}$ is in contrast to the 3D situation in which one has a optical mode starting from the plasma frequency. This is a consequence of the fact that the screening is incomplete in 2D because there are electromagnetic fields in the vacuum surrounding the plane and many-body effects play important role in describing the properties of the 2DES. On the other hand, the longitudinal plasmon mode in the 1DES case is $\omega_{1D}(q_x) \sim q_x \ln(q_x \ell)$, where ℓ is some characteristic length of the system. In these cases, we have assumed that only the lowest subband, for electron motion along the direction perpendicular to the electron sheet, is occupied. This limiting case is achieved when the Bolztmann factor $\exp(-\Delta_{21}/T) \ll 1$, where $\Delta_{21} = \Delta_2 - \Delta_1$ is the energy gap between the lowest (1) and the first-occupied (2) subband, and the occupation of higher subbands is negligible. Otherwise, the multisubband nature of low-dimensional electron systems − hereafter referred as quasi-2D(1D)ES − cannot be discarded when the temperature is comparable with Δ_{21} and population effects of higher subbands cannot be ignored.

In this paper, we address the problem of plasmon spectrum in Q2DES and Q1DES over the surface of liquid helium. We use the many-body dielectric formalism within the

random-phase approximation. In this approach, the mode spectrum is obtained from the roots of a determinantal equation for the dielectric function. At first glance, we note that the multisubband character of these systems allows the existence of transverse modes of plasma oscillations in the direction normal to that of unconfined electron motion.

We adopt a two-subband model where the bare electron-electron interaction is evaluated using subband wave functions found by the variational method for the Q2DES and taken as the harmonic-oscillator functions for the parabolic confinement in the Q1DES. We limit ourselves to the case of low enough temperatures which allows us to disregard the coupling of plasma oscillations with ripplon modes. We do not also consider the possible transition of the electron system to the ordered state where the electron-ripplon interaction can strongly modify the mode spectrum. [4–6]

II. THEORETICAL APPROACH

The main theoretical approach to the study of plasma oscillations in multisubband low-dimensional charge system is based on many-body dielectric formalism using the generalized dielectric function

$$\epsilon_{nn',mm'}(\omega, q) = \delta_{nn'}\delta_{mm'} - V_{nn',mm'}(q)\Pi_{mm'}(\omega, q) \tag{1}$$

where $\Pi_{mm'}(\omega, q)$ is the density-density response function, $\delta_{nn'}$ is the Kronecker symbol and $V_{nn',mm'}(q)$ is the matrix element of Fourier-transformed Coulomb interaction averaged over wave functions of subbands with indices n, n', m, and m' equal to 1, 2, 3.... The dielectric function $\epsilon_{nn',mm'}(\omega, q)$ depends both on the frequency and the wave numbers q for the Q2DES and q_x for the Q1DES.

In the random-phase approximation (RPA), we assume that the electron system responds to external perturbations as a noninteracting system and we take $\Pi_{mm'}(\omega, q) = \Pi^0_{mm'}(\omega, q)$, where the free polarizability function is written as

$$\Pi^0_{mm'}(\omega, q) = \sum_{\mathbf{k},\sigma} \frac{f_0(E_{\mathbf{k}} + \Delta_m) - f_0(E_{\mathbf{k+q}} + \Delta_{m'})}{\hbar\omega + E_{\mathbf{k}} + \Delta_m - E_{\mathbf{k+q}} - \Delta_{m'} + i\delta}. \tag{2}$$

Here $E_{\mathbf{k}} = \hbar^2 k^2/2m$, where m is the electron mass, δ is infinitesimal positive, and σ is spin index. For classical systems, the distribution function $f_0(E_{\mathbf{k}} + \Delta_n) = \exp\left[-\left(E_{\mathbf{k}} + \Delta_n\right)/T\right]$ and is normalized by the condition $\sum_{n,\mathbf{k},\sigma} f_0(E_{\mathbf{k}} + \Delta_n) = N$ where N is the number of particles.

Using the dielectric function given by Eq. (1), Vinter [7] and Das Sarma [8] have studied many-body effects in the degenerate Q2DES. Das Sarma and co-workers [9–11], Hu and O′Connell [12] and Hai *et al.* [13] extended these studies to plasma oscillations in degenerate Q1D multisubband system whereas Sokolov and Studart [14] approach the problem in the classical regime.

The well-known bare electron-electron potential is given by

$$V^{2D}_{nn',mm'}(q) = \int_0^\infty \int_0^\infty dz dz_{'} \psi_n(z)\psi_{n'}(z) v^{2D}(q) \psi_m(z')\psi_{m'}(z'), \tag{3}$$

and

$$V^{1D}_{nn',mm'}(q_x) = \int_{-\infty}^\infty \int_{-\infty}^\infty dy dy' \varphi_n(y)\varphi_{n'}(y) v^{1D}(q_x) \varphi_m(y')\varphi_{m'}(y'), \tag{4}$$

where $v^{2D}(q) = 2\pi\widetilde{e}^2/Sq$, $[v^{1D}(q_x) = 2(\widetilde{e}^2/L_x)K_0(|q_x||y - y'|)$ is the Coulomb potential, S $[L_x]$ the area [length] of the system, and $\psi_n(z)$ $[\varphi_n(y)]$ denotes the n-th subband wave functions for the Q2DES [Q1DES]. Here $\widetilde{e} = [2e^2/(1+\varepsilon)]$, with ε the helium dielectric constant, is the effective charge taking substrate effects into account.

III. PLASMON SPECTRUM

The dispersion relations for collective modes for a multisubband system are found from the roots of the determinantal equation

$$\det|\epsilon_{nn',mm'}(q,\omega)| = 0. \tag{5}$$

In principle, all the subbands should be considered in the above equation. However an useful analytical solution is possible in a two-subband model. In this case, Eq. (5) splits into two independent equations

$$1 - V_{11,11}\Pi^0_{11,11}(\omega, q) = 0 \tag{6a}$$

$$1 - V_{12,12}\left[\Pi^0_{12}(\omega, q) + \Pi^0_{21}(\omega, q)\right] = 0. \tag{6b}$$

Mode coupling appears only if one take into account higher subbands. Equation (6a) describes the longitudinal *intrasubband* plasma oscillations whose dispersion law must coincide with that of 2DES or 1DES with one-subband occupancy system whereas Eq. (6b) gives the dispersion law for transverse *intersubband* oscillations involving transitions from the lowest to the second subband.

A. Q2DES

As it is well-known, SE on helium are trapped in the direction perpendicular to the surface (z direction) by a potential well due to image forces and a holding electric field $E_\perp$. For $E_\perp = 0$, and the image potential $V(z) = -\Lambda_0/z$, where $\Lambda_0 = (e^2/4)(\varepsilon - 1)/(\varepsilon + 1)$, and infinite potential barrier at the interface, the solution of the Schrödinger equation is given by [15–17]

$$\psi_n(z) = \frac{2\kappa_0^{3/2} z}{n^{5/2}} \exp\left(-\frac{\kappa_0 z}{n}\right) L^{(1)}_{n-1}\left(\frac{2\kappa_0 z}{n}\right) \tag{7}$$

where $\kappa_0 = m\Lambda_0/\hbar^2$ ($= 3/(2\left\langle z \right\rangle_0)$, where $\left\langle z \right\rangle_0$ is the mean electron distance from the plane) and $L^{(\alpha)}_n(x)$ are the associated Laguerre polynomials. The energy subband is given by the hydrogen-like spectrum $\Delta_n = \Delta_0/n^2$ where $\Delta_0 = \hbar^2\kappa_0^2/2m$. If the pressing electric field $E_\perp$ is turned on, there is no general analytical solution and we assume trial wave functions corresponding to two lowest subbands ($n = 1$ and 2) of Eq. (7) with variational parameters κ_1 and κ_2: [18,19]

$$\psi_1(z) = 2\kappa_1^{3/2} z \exp\left(-\kappa_1 z\right), \tag{8a}$$

$$\psi_2(z) = \frac{2\sqrt{3}\kappa_2^{5/2}}{\kappa_{12}} \left[1 - \left(\frac{\kappa_1 + \kappa_2}{3}\right) z\right] z \exp\left(-\kappa_2 z\right), \tag{8b}$$

and subband energies:

$$\Delta_1 = \frac{\hbar^2\kappa_1^2}{2m} - \Lambda_0\kappa_1 + \frac{3eE_\perp}{2\kappa_1}, \tag{8c}$$

$$\Delta_2 = \frac{\hbar^2\kappa_2^2}{6m}\left[1 + \frac{6\kappa_2^2}{\kappa_{12}^2}\right] - \frac{\Lambda_0\kappa_2}{2}\left[1 + \frac{2\kappa_2^2 - \kappa_1\kappa_2}{\kappa_{12}^2}\right] + \frac{eE_\perp}{2\kappa_2}\left[1 + \frac{4\kappa_1^2 - \kappa_1\kappa_2 + \kappa_2^2}{\kappa_{12}^2}\right]. \tag{8d}$$

Here $\kappa_{12}^2 = \kappa_1^2 - \kappa_1\kappa_2 + \kappa_2^2$. If we define $\kappa_1(E_\perp) = \eta_1\kappa_0$ and $\kappa_2(E_\perp) = \eta_2\kappa_0$, one can find η_1 and η_2 as the roots of the system of equations given by

$$\eta_1^3 - \eta_1^2 - \left(\frac{\kappa_\perp}{\kappa_0}\right)^3 = 0, \tag{9a}$$

$$\eta_2^3(\eta_1^4 - 2\eta_1^3\eta_2 + 15\eta_1^2\eta_2^2 - 11\eta_1\eta_2^3 + 7\eta_2^4) - \frac{3}{2}\eta_2^2(\eta_1^4 - 4\eta_1^3\eta_2 + 10\eta_1^2\eta_2^2 - 6\eta_1\eta_2^3 + 3\eta_2^4)$$

$$-\left(\frac{\kappa_\perp}{\kappa_0}\right)^3(5\eta_1^4 - 10\eta_1^3\eta_2 + 15\eta_1^2\eta_2^2 - 4\eta_1\eta_2^3 + 2\eta_2^4) = 0 \tag{9b}$$

where $\kappa_\perp = (3meE_\perp/2\hbar^2)^{1/3}$. For $E_\perp = 0$, Eqs. (9a) and (9b) reproduce the results, given by Eq. (7) and respective eigenenergies with $\eta_1 = 1$ and $\eta_2 = 0.5$. The numerical values of the variational parameters as a function of the pressing electric field are plotted in Fig. 1. We observe a rapid increase at low field and an asymptotic linear behavior at larger fields. With these values for η_1 and η_2, we depicted in Fig. 2, the field dependence of the energy of the lowest-subband and the energy gap Δ_{21}.

Using Eqs. (3), (8a) and (8b) one can calculate the values of $V_{11,11}$ and $V_{12,12}$ up to second-order in the parameters $q/\kappa_1 \ll 1$ and $q/(\kappa_1+\kappa_2) \ll 1$ as

$$V_{11,11} = v^{2D}(q)\left[1 - \frac{3q}{4\kappa_1} + \frac{3q^2}{4\kappa_1^2}\right]; \tag{10a}$$

$$V_{12,12} = v^{2D}(q)\alpha(E_\perp)\frac{q}{\kappa_0}\left[1 - \frac{16q}{5(\kappa_1+\kappa_2)} + \frac{7q^2}{(\kappa_1+\kappa_2)^2}\right] \tag{10b}$$

with $\alpha(E_\perp) = 60\eta_1^3\eta_2^5/[(\eta_1+\eta_2)^7(\eta_1^2 - \eta_1\eta_2 + \eta_2^2)]$. The well-behaved form of $\alpha(E_\perp)$ does not influence strongly $V_{12,12}$ because $\alpha(E_\perp = 0) = 0.146$ and $\alpha(E_\perp)$ increases by increasing $E_\perp$

until reaches a maximum $\alpha_{\max} = 0.281$ near $E_\perp = 0.3$ kV/cm and slightly decreases to 0.227 at $E_\perp = 3$ kV/cm.

For the Q2DES, the noninteracting density-density response function, Eq.(2), can be calculated in a straightforward way. The result is

$$\Pi^0_{nn'}(\omega, q) = -\frac{N}{\hbar q u_T Z_n}\left[\exp\left(-\Delta_n/T\right) U\left(\zeta^{(-)}_{nn'}\right) - \exp\left(-\Delta_{n'}/T\right) U\left(\zeta^{(+)}_{nn'}\right)\right] \tag{11}$$

where $\zeta^{(\pm)}_{nn'} = \left[\omega + \left(\Delta_n - \Delta_{n'}\right)/\hbar\right]/q u_T \pm \hbar q/2 m u_T$, with $u_T = \sqrt{2T/m}$ being the thermal velocity and $Z_n = \sum_n \exp\left(-\Delta_n/T\right)$. Similar general structure of $\Pi^0_{nn'}(\omega, q)$ is found in the classical regime of the electron gas in 3D case. [20] The function $U(\zeta)$ is given by the integral

$$U(\zeta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-y^2)}{y - \zeta - i\delta} = -2\exp(-\zeta^2)\int_0^{\zeta}\exp(t^2)dt + i\sqrt{\pi}\exp(-\zeta^2). \tag{12}$$

For $n = n'$ and for small $q \ll (2m\omega/\hbar)^{1/2}$ Eq. (11) can be approximately expressed through $W(\zeta)$, the well-known function in the plasma theory, as [21–24]

$$\Pi^{(0)}_{nn'}(\omega, q) \simeq -\frac{N}{T Z_n} W\left(\frac{\omega}{q u_T}\right)\exp\left(-\frac{\Delta_n}{T}\right). \tag{13}$$

The function $W(\zeta)$ is connected with $U(\zeta)$ by the relation

$$W(\zeta) = -(\partial U/\partial\zeta)/2 = 1 + \zeta U(\zeta).$$

Putting $\omega = \omega(q) - i\gamma_q$, and assuming $\omega(q)/q u_T \gg 1$ and $\left|\omega(q) - \Delta_{21}/\hbar\right|/q u_T \gg 1$, we obtain, in the two-subband model, the dispersion relation for longitudinal intrasubband mode frequencies (ω_l) and damping (γ_l):

$$\omega_l^2(q) = \omega_{2D}^2(q)\left[1 + \frac{3q}{k_D} - \frac{3q}{4\kappa_1}\right], \tag{14a}$$

$$\gamma_l(q) = \sqrt{\pi}\frac{\omega_l^4(q)}{(q u_T)^3}\exp\left[-\frac{\omega_l^2(q)}{(q u_T)^2}\right], \tag{14b}$$

where $\omega_{2D}^2(q) = (2\tilde{e}^2/m a^2)q$ and $k_D = 2\tilde{e}^2/T a^2$ is the 2D Debye wave number and $a = (\pi n_s)^{-1/2}$ is the mean interelectron spacing. One can see, from Eq. (14a), that the first two-terms of the real part of the longitudinal branch are the same as in the classical 2DES

[21–23]. The extra term $(3q/4\kappa_1)$ comes from the effect of the layer thickness, because κ_1 is related to the distance of the electron in the lowest subband from the plane, and lowers the dispersion relation at small wavelengths. From Eq. (14a), we also observe the competition between screening and layer thickness effects on the Q2DES. The Landau damping $\gamma_l(q)$ of the plasma oscillation is practically the same as in the 2DES.

For transverse intersubband modes, we obtain the frequencies (ω_t) and damping (γ_t):

$$\omega_t^2(q) \simeq \omega_{21}^2 + 2\omega_{21}\omega_{2D}^{(sh)} \left[1 + \frac{\left(1 + 3\omega_{2D}^{(sh)}/2\omega_{21}\right) Tq^2}{m\left(\omega_{2D}^{(sh)}\right)^2} - \frac{16q}{\kappa_1 + \kappa_2} \right], \tag{15a}$$

$$\gamma_t(q) \simeq \sqrt{\pi}\frac{\left[\omega_t(q) - \omega_{21}\right]^2}{qu_T} \exp\left[-\frac{\left[\omega_t(q) - \omega_{21}\right]^2}{(qu_T)^2} \right]. \tag{15b}$$

where $\omega_{21} = \Delta_{21}/\hbar$ and $\omega_{2D}^{(sh)} = 2\alpha(E_\perp)\tilde{e}^2/\hbar\kappa_0 a^2$. As one can see the transverse plasmon mode spectrum, given by Eq. (15a), is quite different of the longitudinal branch and has a gap at $q = 0$. The frequency of the characteristic absorption edge is shifted, relatively to the frequency ω_{12} of the intersubband transition, shown in Fig. 2, by $\Delta\omega = \sqrt{\omega_{21}^2 + 2\omega_{21}\omega_{2D}^{(sh)}} - \omega_{21}$, which is the manifestation of the depolarization shift effect in transverse oscillations of the many-body system. [12] The experimental observation of $\Delta\omega$, should be very interesting by evidencing the role of Coulomb effects in the collective electron motion along the z direction. Note that for the 2D plasma parameter $\Gamma = \tilde{e}^2/aT \lesssim 1$, where RPA is formally valid, $\omega_{2D}^{(sh)} \ll \omega_{21}$ and $\Delta\omega \simeq \omega_{2D}^{(sh)}$, *i.e.* the absorption edge is very close to ω_{21} being only slightly shifted to higher frequencies. By increasing q, $\omega_t(q)$ decreases according the last term in brackets in Eq. (15a). However our estimates show that, for $T \sim 0.1 - 1.0$ K and $q \sim 10 - 10^2$ cm^{-1}, the coefficient of the quadratic term is larger than that of linear one. However, in the long wavelength limit, these coefficients are so small that $\omega_t(q) \simeq \omega_{21}\sqrt{1 + 2\omega_{2D}^{(sh)}/\omega_{21}}$. As in the longitudinal mode, $\gamma_t(q)$, given by Eq. (15b), is exponentially small such that $|\gamma_t(q)| \ll \omega_t(q)$.

The absorption edge of the mode $\omega_t(q)$ depends strongly on ω_{21}. For very small $E_\perp$, the experimental values of ω_{21} are close to $3\Delta_0/4\hbar$ and increase linearly with $E_\perp$. [26] For

arbitrary $E_\perp$, ω_{21} is obtained from the gap energy, displayed in Fig. 2, and is in the range of 100 GHz to 1 THz. The polarization shift $\Delta\omega \sim \omega_{sh}^{(2D)} \sim n_s$ for $\Gamma < 1$, even though $|\Delta\omega| \ll \omega_{21}$. For example for $n_s = 10^6$ cm^{-2} and $E_\perp = 0$, we estimate $\omega_{sh}^{(2D)} \sim 100$ MHz $\ll \omega_{21}$. This makes very difficult the direct experimental observation of depolarization shift at this electron density. However, the effect should be observable at higher densities (for instance $n_s \sim 10^8$ cm^{-2}) even though our results can not be quite reliable in this regime since RPA should not be valid in such a strongly correlated classical Q2DES. However, one can hope that the nature of plasma oscillations does not change drastically, at least qualitatively, even in the high density limit ($\Gamma > 1$) and the depolarization shift should be measured for the Q2DES on the helium surface.

B. Q1DES

We now consider plasma oscillations in the Q1DES created along a channel filled with liquid helium. As in previous work, [14,27] we consider a parabolic confinement $U(y) = m\omega_0^2 y^2/2$ with the frequency $\omega_0 = (eE_\perp mR)^{1/2}$, where R is the curvature radius of the liquid in the channel. Typical values of R vary from 10^{-4} to 10^{-3} cm. [28] The spectrum of electron subbands along the y axis is $E_n = \hbar\omega_0(n-1/2)$, $n = 1, 2, 3...$ in addition to the subbands along the z direction. The motion along the x direction (the channel axis) is free. The frequency ω_0 increases with $E_\perp$ achieving 100 GHz at $E_\perp = 3$ kV/cm for $R = 5\times 10^{-4}$ cm. As $\omega_0 \ll \omega_{21}$, the multisubband system in transverse directions can be decoupled and we ignore electron transitions in z direction which are the same as discussed above.

The noninteracting density-density response function was calculated in Ref. [14]. The result was

$$\Pi^0_{nn'}(\omega, q_x) = -\frac{2N\left[\exp\left[-(n-1)\hbar\omega_0/T\right]U\left(\zeta^{(-)}_{nn'}\right) - \exp\left[-(n'-1)\hbar\omega_0/T\right]U\left(\zeta^{(+)}_{nn'}\right)\right]}{\hbar q_x u_T\left[1+\coth\left(\hbar\omega_0/2T\right)\right]} \quad (16)$$

where $\zeta^{(\pm)}_{nn'} = (\omega/q_x u_T)\left[1+(\omega_0/\omega)(n-n')\right] \pm \hbar q_x/2mu_T$. For $\hbar\omega_0 \gg T$, when only the lowest subband ($n = 1$) is occupied, the expression for the response function is greatly simplified

yielding $\Pi^0_{nn'}(\omega, q_x) = -(N/T)W(\omega/q_x u_T)$.

Using the wave functions of the two lowest subbands ($n=1$ and $n=2$)

$$\varphi_1(y) = \frac{1}{\pi^{1/4} y_0^{1/2}} \exp\left(-\frac{y^2}{2y_0^2}\right); \qquad \varphi_2(y) = \frac{\sqrt{2}}{\pi^{1/4} y_0^{3/2}} y \exp\left(-\frac{y^2}{2y_0^2}\right),$$

where $y_0 = (\hbar/m\omega_0)^{1/2}$, we obtain the matrix elements of Coulomb interaction from Eq. (4):

$$V^{1D}_{11,11}(q_x) = \frac{\tilde{e}^2}{L_x} \exp\left(\frac{q_x^2 y_0^2}{4}\right) K_0\left(\frac{q_x^2 y_0^2}{4}\right) \simeq \frac{\tilde{e}^2}{L_x} \ln\frac{1}{|q_x y_0|^2} \text{ for } |q_x y_0| \ll 1 \tag{17a}$$

and

$$V^{1D}_{12,12}(q_x) = \frac{\tilde{e}^2}{2L_x} \exp\left(\frac{q_x^2 y_0^2}{4}\right) \left[K_0\left(\frac{q_x^2 y_0^2}{4}\right) - \frac{\sqrt{\pi}}{\sqrt{2}|q_x y_0|} W_{-1,0}\left(\frac{q_x^2 y_0^2}{2}\right)\right] \tag{17b}$$

$$\simeq \frac{\tilde{e}^2}{L_x}\left[1 - \frac{q_x^2 y_0^2}{2} \ln\frac{1}{|q_x y_0|}\right] \text{ for } |q_x y_0| \ll 1.$$

where $W_{\alpha,\beta}(x)$ is Whittacker function.

Using Eqs. (5), (6a), and (17a), taking $\omega = \omega(q_x) - i\gamma_q$, and assuming $\omega(q_x)/q_x u_T \gg 1$ and $|\omega(q_x) - \omega_0|/q_x u_T \gg 1$ we obtain the dispersion relation of the longitudinal intrasubband modes in the long wavelength limit, $|q_x y_0| \ll 1$, as

$$\omega_l(q_x) = \frac{2\tilde{e}^2 q_x^2}{m\ell} \ln\frac{1}{|q_x y_0|} \exp\left(\frac{q_x^2 y_0^2}{4}\right)\left[1 + \frac{3T\ell}{2\tilde{e}^2} \ln^{-1}\frac{1}{|q_x y_0|}\right], \tag{18a}$$

$$\gamma_l(q_x) = \sqrt{\pi}\frac{\omega_l^4(q_x)}{(q_x u_T)^3} \exp\left[-\frac{\omega_l^2(q_x)}{(q u_T)^2}\right], \tag{18b}$$

where $\ell \simeq n_l^{-1} = (N/L_x)^{-1}$ is the mean interelectron distance along the channel.

The longitudinal spectrum mode, given by Eq. (18a), has the same structure of the obtained previously in Ref. [14] and in Refs. [29,30] where a quasi-crystalline approximation was employed. However we found an additional second term in brackets, which should be quite small for reasonable values of T and ℓ. Note also that condition $\omega/q_x u_T \gg 1$ assumed here is equivalent to $T \ll e^2/\ell$ in the quasicrystalline approximation. It worth emphasizes that the present result was obtained within RPA which is valid in the opposite

limit $T \gg e^2/\ell$. Our conclusion is that the plasmon spectrum in the classical Q1DES has little dependence on the plasma parameter and RPA results should be probably correct in wide range of electron densities.

The transverse branch of collective excitations is rather interesting. Following the same steps as before, we arrive to

$$\omega_t^2(q_x) = \omega_0^2 - \frac{\widetilde{e}^2 q_x^2}{m\ell} \ln \frac{1}{|q_x y_0|} \tag{19a}$$

$$+2\omega_0\omega_{1D}^{(sh)} \left[1 + \left(\frac{\left(1 + 3\omega_{1D}^{(sh)}/2\omega_0\right) T}{m\left(\omega_{1D}^{(sh)}\right)^2} + \frac{\left(1 + \omega_{1D}^{(sh)}/\omega_0\right)\hbar}{2m\omega_{1D}^{(sh)}} \right) q_x^2 \right],$$

$$\gamma_t(q_x) = \sqrt{\pi}\frac{[\omega_t(q_x) - \omega_{21}]^2}{q_x u_t} \exp\left[-\frac{[\omega_t(q_x) - \omega_{21}]^2}{(q_x u_t)^2} \right] \tag{19b}$$

Here $\omega_{1D}^{(sh)} = \widetilde{e}^2/\hbar\ell$. The first two terms in Eq. (19a) correspond to the result obtained in the quasicrystalline approximation [29,30] if y_0 is replaced by ℓ in the logarithmic factor. The next term is the depolarization shift correction increasing the absorption edge frequency by $\Delta\omega = \sqrt{\omega_0^2 + 2\omega_0\omega_{1D}^{(sh)}} - \omega_0 \simeq \omega_{1D}^{(sh)}$ when $\omega_{1D}^{(sh)} \ll \omega_0$. One can see that the instability of the transverse mode ($\omega_t^2(q_x) < 0$) in the limit of zero confinement ($\omega_0 = 0$) is still manifested in our treatment. We call the attention that we found a quite different result in our previous work [14] because we used an approximate expression $V_{12,12}^{1D}(q_x) \simeq \widetilde{e}^2/L_x$ considered in Ref. [12]. One estimative is that the polarization shift correction should be quite small for $n_l \sim 10^2 - 10^3$ cm^{-1} and $T \simeq 10^{-1} - 1$ K such that $e^2/\ell < T$. For instance, $\omega_{1D}^{(sh)} \simeq 10$ GHz for $E_\perp = 3$ kV/cm and $n_l = 10^2$ cm^{-1} whereas $\omega_0 = 100$ GHz at $R = 5 \times 10^{-4}$ cm. However, this density range can not be achieved in experimental conditions. For higher densities, $\Delta\omega$ should be of the same order of ω_0 and the polarization shift should be observed even our results are based on the RPA.

IV. CONCLUDING REMARKS

In the present work, we have used the many-body dielectric formalism to calculate the spectrum of plasma oscillations for the classical Q2DES and Q1DES formed on the liquid helium surface. We have obtained the general expression for the density-density Q2D and Q1D response functions for any frequency and wave number within the RPA. The results are valid at low temperatures since we have used a two-subband model in which only the lowest subbands of the motion in the direction normal to the electron layer (Q2D) and of the motion in direction across the conducting channel (Q1D) are occupied. The plasma dispersion relations were found from the zeros of the determinantal equation for the generalized multisubband dielectric functions. We have obtained corrections to the gapless longitudinal modes, beyond the $q^{1/2}$-behavior in the Q2DES and sound-like behavior, within logarithmic accuracy, in the Q1DES. The intersubband transverse collective frequency is higher than the corresponding single-particle excitation frequency both in Q2DES and Q1DES. The absorption edge frequencies are increased by the depolarization shift which can be large at high densities. In this connection, the experimental study of intersubband transition in low-dimensional electron systems over liquid helium seems to be attractive, because of the accessibility of wide range of charge concentrations and low temperatures, to observe collective effects on spectroscopic transitions. [26]

We conclude by pointing out some limitations of our approach. The results are based on the RPA, which works quite well at small values of the plasma parameter. We know that RPA results become worse as the dimensionality is reduced, but we do not know how to go beyond RPA in a controlled way mainly in the Q1DES. We are, however, encouraged by the good agreement of our RPA results for the mode spectrum and those obtained in the quasi-crystalline approximation that is valid in the opposite limit of high plasma parameter. Other fact is the excellent agreement between the RPA theory and experiment on collective excitations in semiconductor quantum wells [31] and wires [32]. Our use of a two-subband model can be and should be improved in more realistic calculations [33]. But we do not

expect the correction of including other subbands to be qualitatively significant though at low temperatures.

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FIGURE CAPTIONS

Fig. 1. Variational parameters η_1 (straight line) and η_2 (dashed line) as a function of the pressing electric field $E_\perp$ evaluated numerically from Eqs. (9a) and (9b).

Fig. 2. Lowest-subband energy (straight line) and energy gap Δ_{21} (dashed line) of single-electron spectroscopic excitation of the Q2DES as a function of the pressing electric field $E_\perp$.

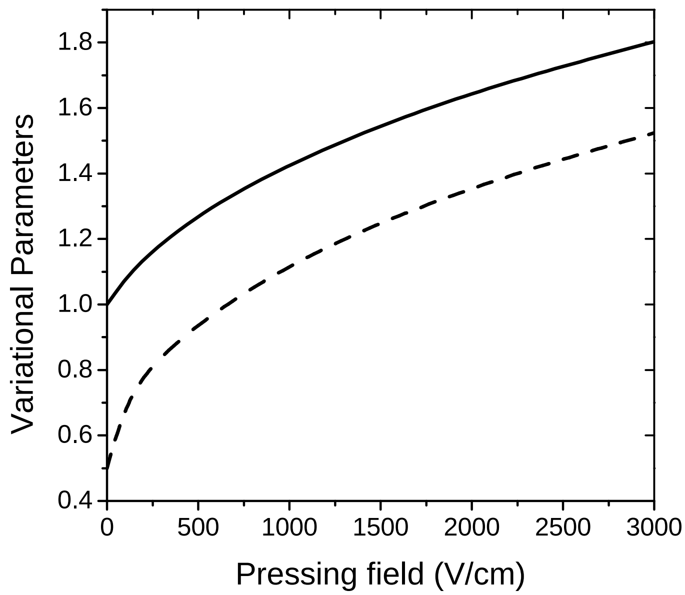

Fig. 1.
Sokolov and Studart,
"Plasma dispersion..."

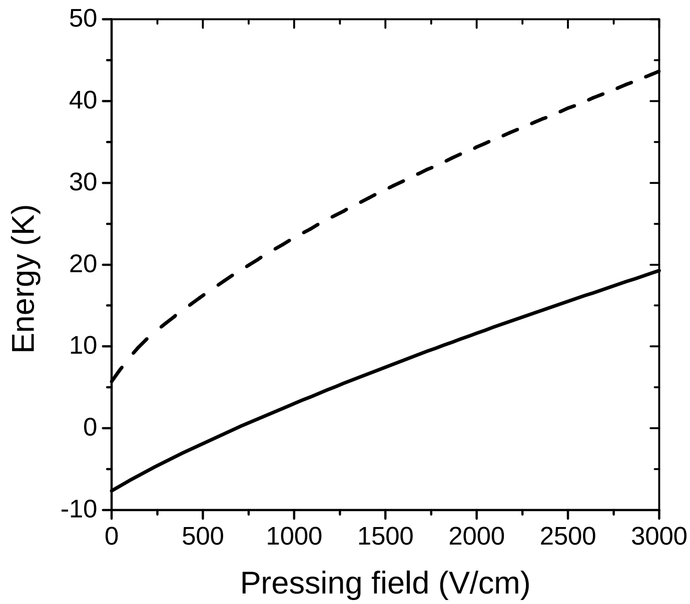

Fig. 2.
Sokolov and Studart,
"Plasma dispersion..."